EXPANSIONS, INEQUALITIES AND APPROXIMATIONS. ON THE OCCASION OF GAVIN BROWN'S 65TH BIRTHDAY

(University of Sydney, March 5–6, 2007)

List of Talks

Jim Byrnes Prometheus Inc (USA)

Shapiro Sequences, Reed-Muller Codes, and Functional Equations

We show how the Shapiro sequences can be thought of as energy spreading second order Reed-Muller codewords, i.e., members of $RM(2, m)$, and in fact why the (rows of the) Welti (PONS) matrix is a coset of the subgroup $RM(1, m)$ of $RM(2, m)$. This also leads to interesting questions regarding functional equations satisfied by generating functions of blocks of binary digits.

Feng Dai University of Alberta (Canada)

Kolmogorov and linear n-widths for the Sobolev classes on the unit sphere.

This talk is a joint work with Gavin Brown, conducted when I was a student at the University of Sydney. The main purpose of our research is to estimate Kolmogorov n-widths $d_n(B_p^r, L^q)$ and linear n-widths $\delta_n(B_p^r, L^q)$, ($1 \le q \le \infty$) of Sobolev's classes B_p^r, ($r > 0$, $1 \le p \le \infty$) on the unit sphere $\mathbf{S}^{d-1}$ of the d-dimensional Euclidean space $\mathbf{R}^d$. For part of $(p, q) \in [1, \infty] \times [1, \infty]$, sharp orders of $d_n(B_p^r, L^q)$ or $\delta_n(B_p^r, L^q)$ were previously known. In our work, we obtained the sharp orders of $d_n(B_p^r, L^q)$ and $\delta_n(B_p^r, L^q)$ for all the remaining cases of (p, q). Our proof is based on positive cubature formulas and Marcinkiewicz-Zygmund (MZ) inequalities for the spherical polynomials on $\mathbf{S}^{d-1}$. Our work also reveals a close relation between positive cubature formulas and MZ inequalities on $\mathbf{S}^{d-1}$.

Tony Dooley University of New South Wales

G-measures

In the late 1970s, Gavin observed that Riesz product measures based on powers of 3 are ergodic for the action of the triadic rationals. This caused us to examine the role of Riesz product type constructions in ergodic theory and led us to explore the theory of G-measures.

I will give an overview of our work, and show how it has now led to a structure theorem for all non-singular ergodic dynamical systems up to orbit equivalence: Hamachi and I recently proved that every such system is orbit equivalent to a uniquely ergodic G-measure on a Bratteli-Vershik space, realised as an induced transformation on a closed set of an infinite product space.

Ian Doust University of New South Wales

Balanced matrices and functions

There is an interesting family of norms on $\mathbb{R}^n$ given by

$$\|(x_1, \ldots, x_n)\|_{1,k} = \max_{|J|=k} \sum_{j \in J} |x_j|$$

where the maximum is taken over all k element subsets of $\{1, \ldots, n\}$. Let A be an $n \times n$ matrix with rows $\mathbf{r}_1, \ldots, \mathbf{r}_n$ and columns $\mathbf{c}_1, \ldots \mathbf{c}_n$. We say that A is k-balanced if $\|\mathbf{r}_i\|_{1,k} = R$ for all i and $\|\mathbf{c}_j\|_{1,k} = C$ for all j. A little experimentation shows that R and C can be different. (The first interesting case occurs with $n = 4$ and $k = 3$.) Finding the optimal inequalities relating R and C has proven to be a challenge and there are still many open problems. The concept of a balanced function is defined analogously, now integrating of sets of a fixed measure. Our state of knowledge here is much more limited, especially if one restricts one's attention to continuous functions. The big open question is whether a continuous balanced function on $[0, 1] \times [0, 1]$ exists which has a nontrivial ratio R/C? This is joint hobby mathematics with Richard Aron (Kent State) and Nigel Kalton (Columbia, Missouri).

Bill Moran   University of Melbourne

Measures, uniform distribution, and inequalities old and new

I will discuss some old results of Gavin and myself, and some more recent ones. I will give a potted history of our early years in working on measure algebras in Liverpool, leading up to our first skirmish with the kind of inequalities that Gavin has continued to work on. Our more recent work on Schmidt's conjecture will also be exposed briefly before I turn to some recent work of mine motivated by problems in engineering.

Ferenc Móricz   University of Szeged (Hungary)

Absolutely convergent Fourier series and classical function classes

This is a survey talk on the recent progress in the study of the continuity and smoothness properties of a function f with absolutely convergent Fourier series. We give best possible sufficient conditions in terms of the Fourier coefficients of f in order that f belong either to one of the Lipschitz classes $\mathrm{Lip}(\alpha)$ and lip(α) for some $0 < \alpha \le 1$, or to one of the Zygmund classes Zyg(α) and zyg(α) for some $0 < \alpha \le 2$. The termwise differentiation of Fourier series is also discussed. Our theorems generalize some of those proved by R.P. Boas Jr., J. Németh and R.E.A.C. Paley.

Jacques Peyrière   Université Paris-Sud (France)

On the multifractal formalism

An early mathematical treatment of the multifractal analysis of measures appeared in a joint article with G. Brown and G. Michon in 1992. An account of further developments is given: the Olsen formalism, the last version of its justification and some extension. In particular, this allows to extend the Besicovitch theorem on linear sets defined in terms of upper frequency of digits in base 2 expansions to an arbitrary base.

Alf van der Poorten   CeNTRe for Number Theory Research (Australia)

An awful problem about integers in base four

Back in the early eighties, John Loxton and I had our attention captured by a question of Brown and Moran concerning a construction seemingly relevant to work that eventually led to their 1984 paper with Tijdeman: "Riesz products are basic measures". In brief, one may of course write the integers in base four using $0, \pm 1$, and 2 as the four digits. Can every integer be expressed as a quotient of integers requiring just the three digits 0, and ± 1? Such questions are notoriously slippery and John and I therefore remain inordinately proud of the clever tricks we eventually recalled, after far too much effort, to settle the issue. I will tell the story of our travails.

Kun-Yang Wang   Beijing Normal University (China)

A survey of my joint research with Gavin Brown

I have been collaborating with Gavin Brown since 1990. Our research mainly concerns the positivity of trigonometric sums and Jacobi polynomial sums. It also concerns multiple trigonometric sums and the convergence of the linear means of multiple Fourier series and Fourier-Laplace series. The present report is a survey on our joint results.

Qinghe Yin   Australian National University

β-transformations, iterated function systems, and Cantor-type sets

β-transformations and β-expansions were first introduced by Renyi (1957) and further explored by Parry (1961). Given $\beta > 1$, the β-transformation $T_\beta : [0, 1) \to [0, 1)$ is defined by $T_\beta x = \beta x \pmod 1$. It possesses an absolutely continuous invariant measure and is ergodic. The β-expansion of $x \in [0,1)$ is determined by T_β. This introduces a symbolic dynamical system Σ_β with one-sided shift map. An iterated function system (IFS) is an $(n+1)$-tuple $(X; f_0, \dots, f_{n-1})$, where X is a compact metric space and each f_i is a contractive map. The theory of IFS's was first explored by Hutchinson (1981). Many important fractals such as the Sierpinski triangle and the Cantor middle-third set appear as attractors of certain IFS's. We define the β-attractor E_β for an IFS as a compact subset, determined by Σ_β, of the attractor of the IFS. In the case that all f_i are similitudes with contractive ratio $0 < r < 1$ and with a disjoint condition, the Hausdorff dimension of E_β is given by

$$\dim(E_\beta) = \frac{\log \beta}{-\log r}.$$

As an application of this result, we compute the Hausdorff dimension for Cantor-type sets constructed from β-expansions with $\beta > 2$. This is a joint work with John Hutchinson.