MATH 402 Homework 7

Due Friday October 27, 2017

Exercise 1.

a. Show that the symmetry group of a regular n-gon is finite.

b. Show that the symmetry group of a regular n-gon is the dihedral group D_n.

c. Draw a figure whose symmetry group is cyclic and not dihedral.

Exercise 2. Recall the notation from frieze groups: $\tau = T_v$ is the fundamental translation, γ is a glide reflection with translation vector $\frac{1}{2}\vec{v}$ and line of reflection the midline m, H is a half-rotation about a point on the midline, r_ℓ is a perpendicular reflection (that is, reflection across a line ℓ which is perpendicular to the direction of translation $\vec{v}$).

a. Suppose that G is a frieze group with fundamental translation τ, and suppose that G contains at least one perpendicular reflection, r_ℓ. Show that all other perpendicular reflections $r_{\ell'} \in G$ can be written in as a composition of copies of τ (or τ^{-1}) and r_ℓ. That is, all perpendicular reflections are *generated by* r_ℓ and τ.

b. Prove that the group generated by γ and H is equal to the group generated by τ, γ, and H.
Hint: To prove that two sets A and B are equal, we can show that $A \subset B$ and $B \subset A$. For A and B the sets underlying the groups above, one of the containments is easy. Which one? How can you prove the other one?

c. Do exercise 6.2.13 from the book. (Proofs not required.)

Exercise 3.

a. Sketch the lattices spanned by the following pairs of vectors, and specify which of the five types of lattices each pair will generate.
   i. $\vec{v} = (2, 0), \vec{w} = (2, 4)$.
   ii. $\vec{v} = (1, \sqrt{3}), \vec{w} = (1, -\sqrt{3})$.
   iii. $\vec{v} = (-1, 1), \vec{w} = (1, 1)$.

b. Show that the transformation $f(x, y) = (-x, y + 1)$ is a glide reflection.

c. *[Bonus]* Show that the symmetry group generated by f and the translation $T(x, y) = (x+1, y+1)$ must be a wall-paper group. Which lattice type will this group have? (To answer this question, identify the vectors $\vec{v}$ and $\vec{w}$.)

d. Show that the symmetry group generated by the glide reflection f and the translation $T(x, y) = (x, y+1)$ is not a wall-paper group.

Remember that in addition to the points assigned to each question, you will receive up to five further points for neatness and organization.