

MATH 402 Homework 9

Due Friday November 17, 2017

Exercise 1.

- Prove that in a Saccheri quadrilateral, the summit is always larger than the base. (Hint: divide the Saccheri quadrilateral into two Lambert quadrilaterals, as we did in class. What do you know about opposite sides of a Lambert quadrilateral?)
- Let $ABCD$ and $A'B'C'D'$ be two Saccheri quadrilaterals, with bases \overline{AB} and $\overline{A'B'}$ and summits \overline{CD} and $\overline{C'D'}$. Suppose that summits are congruent, and that the summit angles are also congruent. Prove that the Saccheri quadrilaterals themselves must be congruent: that is, prove that the bases are congruent, and that the sides are congruent.

Hint: suppose (towards a contradiction) that the side \overline{AD} is longer than the side $\overline{A'D'}$. Then you can find a point D'' on \overline{AD} so that $AD'' = A'D'$... Try to construct a rectangle, which will give you the contradiction, because we know there are no rectangles in hyperbolic geometry.

- On the other hand, suppose that instead you know that the bases are congruent and that the summit angles are congruent, but you don't know anything about the summits. Prove that once again, the Saccheri quadrilaterals are congruent: that is, the summits are congruent after all, and the sides are also congruent.

Hint: once again, assume towards a contradiction that one of the quadrilaterals has longer sides. Choose points on those longer sides, and try to construct a quadrilateral that has angle sum 360° to get your contradiction.

Exercise 2.

- Let ℓ and m be two parallel lines. Recall that for a point P on ℓ , the distance from P to m is determined by drawing the perpendicular n from P to m , and by measuring the distance from P to the point of intersection.

Show that given three points A, B, C on ℓ , they cannot all have the same distance to m . (Hint: you can assume that B lies between A and C at ℓ . If the distances are all the same, show that you get two Saccheri quadrilaterals. What do you know about the angles at B ?)

- Let ℓ be a line and m a limiting parallel to ℓ through the point P . Suppose that P' is another point on m , which is "closer" to the omega-point of the lines ℓ and m than P is. (It's in quotation marks, because of course P and P' are both infinitely far from the omega-point, but it should still tell you how to draw the picture.)

Prove that the distance from P' to ℓ is smaller than the distance from P to ℓ . I can think of two ways to prove this result. You can use either method (or come up with a different one); bonus marks if you do more than one!

(1) What is the angle of parallelism of m to ℓ at P and at P' ? It follows from Worksheet 8 (you need to justify this) that if you can show that the angle of parallelism is *larger* at P' , then the distance from P' to ℓ is *smaller*.

(2) Let n' be the perpendicular from P' to ℓ . Draw the perpendicular from P to n' , intersecting n' at some point X . Where does X lie on this line? Then use facts about Lambert quadrilaterals.

Exercise 3.

- Prove that if a polygon is made up of subtriangles, the defect of the polygon is the sum of the defects of each of the subtriangles.
- Use this to show that if two triangles are equivalent, they must have the same defect.

Remember that in addition to the points assigned to each question, you will receive up to five further points for neatness and organization.