MATH 402 Practice questions

Friday 2 November, 2018

Exercise 1. Hyperbolic geometry: Let ℓ and m be two Klein lines which are parallel but not limiting parallel. Prove that there is a unique Klein line n which is perpendicular to both ℓ and m.

Exercise 2. Checking something is an isometry:

(a) What is the definition of an isometry $f : \mathbb{R}^2 \to \mathbb{R}^2$?

(b) Review the proof that any isometry can be written as a composition of at most three reflections.

(c) For each of the following functions, is it an isometry? Prove or disprove.

$$\begin{aligned} f(x,y) &= (y,x) \\ g(x,y) &= (x+y, y+a) \\ h(x,y) &= (x+a, y+b) \\ j(x,y) &= (-y, x) \end{aligned}$$

Exercise 3. Using the classification of isometries to identify isometries:
Consider the following functions of the Euclidean plane. For each, indicate in the table whether it is possible that the function is a reflection, (non-identity) rotation, (non-identity) translation, glide reflection (with non-zero displacement vector), or the identity. For this problem, assume that ℓ and m are two lines and O is a point on ℓ.

	Reflection	Rotation	Translation	Glide reflection	Identity
$f = r_\ell \circ r_m \circ r_\ell$					
An isometry f which satisfies $f(O) = O$					
An even isometry					
The composition of a glide reflection and a translation					
The function $f(x, y) = (2x, y)$					
An isometry f which has ℓ as its *only* invariant line					
An isometry f which satisfies $f^3 = \text{id}$					
An isometry f which can be represented by a 2×2 matrix					
An isometry f which has no fixed points					
An isometry f which is the square of a glide reflection					