MATH 402 Homework 6

Due Friday October 19, 2018

Exercise 1. *[8 pts]* Prove the following theorem:

Theorem 1. *Suppose that f and g are two isometries which agree on three non-collinear points A, B, C. Prove that $f(P) = g(P)$ for* all *points P.*

Exercise 2. *[8 pts]* Prove that an isometry preserves circles: i.e. if f is an isometry, and c is a circle with radius r and centre O, then f maps c to the circle c' of radius r and centre $f(O)$.

Exercise 3. Recall that a *reflection* r is defined as an isometry which has two fixed points, and which is not the identity.

a. *[6 pts]* Define what it means for a set S to be *fixed* by r. Define what it means for S to be *invariant* under r.

b. *[10 pts]* Recall that we proved that the reflection fixes the entire line ℓ determined by these two points, and we denoted this reflection by $r = r_\ell$. Now prove that the invariant lines of r_ℓ are exactly the line ℓ and the lines m which are perpendicular to ℓ. *That is, suppose n is a line which is not equal to ℓ. Prove that n is invariant if and only if n is perpendicular to ℓ.*

Exercise 4. a. *[3 pts]* Let $T = r_{\ell_2} \circ r_{\ell_1}$ be a translation, with displacement vector v. Prove that the inverse of T is also a translation, given by $r_{\ell_1} \circ r_{\ell_2}$ and having displacement vector $-v$.

b. *[3 pts]* Let T_1 and T_2 be two translations, with displacement vectors v_1 and v_2 respectively. Prove that $T_1 \circ T_2$ is again a translation. What is its displacement vector?

c. *[3 pts]* Show that composition of translations *commutes*: that is, that $T_1 \circ T_2$ is equal to $T_2 \circ T_1$. Is this true for reflections? Prove or provide a counter-example.

d. *[4 pts]* Does the set of translations form a group?

Remember that in addition to the points assigned to each question, you will receive up to five further points for neatness and organization.