Name:

netid.:

Math 402: Exam 2

Fall semester 2018

- Do not forget to write your name and netid on top of this page.
- No notes, books, calculators, or other exam aids are allowed. You may use a ruler and colored pens or pencils if you wish.
- Turn your cell phones off and put them away. **No use of cell phones** or other communication devices during the exam is allowed.
- Write your answers **clearly and fully** on the sheets provided. If you need additional paper, raise your hand.
- **Do not tear pages** off of this exam. Doing so will be considered cheating.
- The exam consists of 5 problems and 6 pages. Check that your exam is complete.
- You have **50 minutes** to complete the exam.

Good luck!!

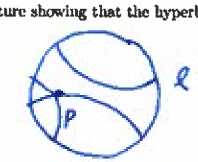
Problem	1	2	3	4	5	Σ
Total possible	18	12	20	15	35	100
Your points						

Problem 1: $(5 + 3 + 10 = 18$ Points)

(a) State the hyperbolic parallel postulate.

Given a line ℓ and a point $P \notin \ell$, $\exists$ at least two lines through P parallel to ℓ.

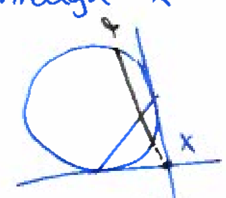
(b) Draw a picture showing that the hyperbolic parallel postulate holds in the Poincaré model.



(c) Define what it means for a Klein line ℓ to be *perpendicular* to a Klein line m. (Assume that m is not a diameter.) Draw a picture illustrating the definition.

Let m have $X = \text{pole}(m)$ (a point outside the disk).

ℓ is perpendicular to m $\iff$ the Euclidean line corresponding to ℓ passes through X



Problem 2: (4 + 8 = 12 Points)

(a) State the definition of an isometry on $\mathbb{R}^2$.

$f : \mathbb{R}^2 \to \mathbb{R}^2$ s.t.

$f(A)f(B) = AB \quad \forall \; A, B \in \mathbb{R}^2$

(b) Consider the function f given by

$$f(x, y) = (x + 2, -y).$$

Is f an isometry? Prove or disprove.

let $A = (x, y)$, $B = (z, w)$

$AB = \sqrt{(x-z)^2 + (y-w)^2}$

$f(A) = (x+2, -y)$, $f(B) = (z+2, -w)$

$f(A)f(B) = \sqrt{((x+2) - (z+2))^2 + ((-y) - (-w))^2}$

$= \sqrt{(x-z)^2 + (w-y)^2} = AB$

Problem 3: (10 + 10 = 20 Points) Choose **two** of the following three problems to solve. (If you attempt all three, your best two scores will be counted. But don't spend too long on this problem until you've finished the rest of the exam!)

(a) Let R be rotation around the point P by some angle ϕ. Let m be a line passing through P. What is the isometry $r_m \circ R \circ r_m$? (Be as specific as you can.)

choose ℓ s.t. $\ell \cap m = P$ with angle $\frac{1}{2}\phi$

so $R = r_\ell \circ r_m$

$\Rightarrow r_m \circ R \circ r_m = r_m \circ r_\ell \circ r_m \circ r_m = r_m \circ r_\ell$

$= R^{-1} = R_{P, -\phi}$.

(b) Let ℓ be the perpendicular to m at P. Prove that $r_\ell \circ r_m = r_m \circ r_\ell$.

$r_\ell \circ r_m \circ r_\ell = r_{r_\ell(m)} = r_m$ since $\ell \perp m$.

$\Rightarrow r_\ell \circ r_m = r_m \circ r_\ell$

(c) Let G be a glide reflection with non-zero displacement vector $\vec{v}$. Prove that G has no fixed points.

$G = r_\ell \circ T_{\vec{v}} = T_{\vec{v}} \circ r_\ell$

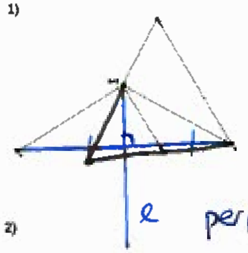
$\Rightarrow G \circ G = T_{\vec{v}} \circ r_\ell \circ r_\ell \circ T_{\vec{v}}$

$= T_{\vec{v}} \circ T_{\vec{v}}$

$= T_{2\vec{v}}$, which has no fixed points.

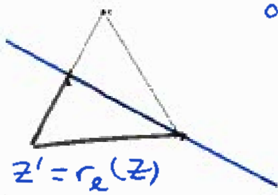
Problem 4: (15 Points) In the picture below, $\triangle ABC$ and $\triangle XYZ$ are two congruent triangles with a common vertex $A = X$. Describe a sequence of two reflections that takes $\triangle XYZ$ to $\triangle ABC$. On the first picture below, show your first line of reflection. In the second picture, draw the image of $\triangle XYZ$ under the first reflection, and draw the second line of reflection. In the third picture, draw the final image of $\triangle XYZ$. Justify your work as you go—don't just draw the lines, explain how you found them.

1)



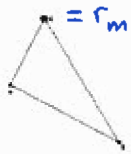
ℓ perpendicular bisector of $\overline{YB}$

2)



$Z' = r_\ell(Z)$ $\quad m = \overleftrightarrow{AB}$

3)



$= r_m \circ r_\ell(Z)$

Problem 5: (35 Points) Consider the following functions of the Euclidean plane. For each, indicate in the table whether it is possible that the function is a reflection, (non-identity) rotation, (non-identity) translation, glide reflection (with non-zero displacement vector), or the identity. For this problem, assume that ℓ and m are two lines that intersect at a point O with an angle of $\alpha = 30°$.

	Reflection	Rotation	Translation	Glide reflection	Identity
An isometry f which has m and ℓ as fixed lines					✓
An isometry f which has ℓ as its *only* invariant line				✓	
An isometry f which is the composition of two rotations		✓	✓		✓
An isometry f which is its own inverse : $f^2 = \mathrm{id}$	✓	✓			✓
An isometry f such that $f(\ell) = m$	✓	✓		✓	
An isometry f which fixes two points A and B	✓				✓
An isometry f which can be written as the composition of 7 reflections	✓			✓	