# MATH 595 Tuesday 13 February

## Čech cohomology

(1) **Chapter III, Exercise 4.1.**

Let $f : X \to Y$ be an affine morphism of noetherian separated schemes. Show that for any quasi-coherent sheaf $\mathscr{F}$ on $X$, and for any $i \geq 0$

$$H^i(X, \mathscr{F}) \cong H^i(Y, f_*\mathscr{F}).$$

(2) **Chapter III, Exercise 4.3.**

Let $U = \mathbb{A}^2_k \setminus \{(0,0)\}$, with coordinates $x$ and $y$. Use a suitable open affine cover of $U$ to show that $H^1(U, \mathcal{O}_U)$ is isomorphic to the $k$-vector space spanned by $\{x^i y^j | i, j < 0\}$. (In particular, it is infinite-dimensional.)

(3) **Chapter III, Exercise 4.5.**

For $(X, \mathcal{O}_X)$ a ringed space, $\mathrm{Pic}_X$ is the group of isomorphism classes of invertible sheaves. Prove that

$$\mathrm{Pic}_X \cong H^1(X, \mathcal{O}_X^*).$$

Hint: for this exercise, use the fact that $H^1(X, \mathscr{F})$ is the colimit of the Čech cohomology groups $\check{H}^1(\mathcal{U}, \mathscr{F})$.

Define a map in one direction as follows: given a line bundle $\mathscr{L}$, choose a cover $\mathcal{U}\{U_i\}$ and trivializations $\phi_i : \mathscr{L}_{|U_i} \to \mathcal{O}_{U_i}$. Use these to construct an element of $\check{H}^1(\mathcal{U}, \mathcal{O}_X^*)$, and hence of $H^1(X, \mathcal{O}_X^*)$. Show that this element is independent of your choices. What is the inverse map?