Sketching the graph of a function in two variables

Let $f(x, y) = y^2$.

- Draw the level sets in the xy-plane for a few choices of $z = k$.
- Draw the contour map in the xy-plane.
- Sketch the surface in $\mathbb{R}^3$.

Compare with your neighbour. Do your pictures agree?

(a) Yes.

(b) No, but I'm pretty sure I'm right.

(c) No, and I don't know who is right.

Symmetry about an axis

Which of the following functions are symmetric about the y-axis?

- $f(x, y, z) = x^2 + y^2 + z^2$;
- $g(x, y, z) = x + 2y^2 + z$;
- $h(x, y, z) = 3x^2 + y + 3z^2$.

(a) None of them.

(b) Only f.

(c) All of them.

(d) f and h.

Correct answer: (d)

Sketching a quadric surface

Consider the equation

$$x^2 + 6x + 2y^2 + 2z^2 + 7 = 0.$$

Note that (by completing the square and dividing both sides by 2) we can rewrite it as

$$\frac{(x+3)^2}{2} + y^2 + z^2 = 1.$$

The level sets $z = k$ are mostly

(a) parabolas or circles;

(b) ellipses or empty;

(c) lines;

(d) I'm still trying to remember what "completing the square" means.

Correct answer: (b), but (d) is also understandable. Remind yourself, or ask for help!

Sketching a quadric surface

Consider the equation

$$\frac{(x+3)^2}{2} + y^2 + z^2 = 1.$$

Think about the traces $x = k$ and $y = k$ of this surface. Try to sketch the surface. Does your picture agree with your neighbour's?

(a) I'm not done.

(b) Yes.

(c) No.

If your pictures don't agree, try to figure out which is right. Change your answer to (b) once you have decided.

Example: can you find a good choice of δ?

Consider the function

$$E(h) = \begin{cases} 0, & h \notin \mathbb{Q}; \\ h, & h \in \mathbb{Q}. \end{cases}$$

Suppose you are given an error tolerance/challenge of $\epsilon > 0$.
Which of the following is a good response δ?

(a) It might not be possible to find δ if ϵ is very small.

(b) Anything will work for δ.

(c) $\delta = \frac{1}{\epsilon}$.

(d) $\delta = \epsilon$.

Correct answer: (d)