# Last time: absolute maxima and minima

Is there a point on the graph $z = \sqrt{x^2 + y^2}$ that's closest to the point $P = (4, 2, 0)$? Furthest?

(a) There is a closest point and a furthest point.

(b) There is a closest point but no furthest point.

(c) There is a furthest point but no closest point.

(d) There is neither a furthest point nor a closest point.

(e) I don't know.

Correct answer: (b)

# Recall: the Extreme Value Theorem

Let $f : D \to \mathbb{R}$ be a continuous function on $D$, which is closed and bounded. Then $f$ attains a maximum value on $D$ at some point $P \in D$, and either

- $P$ is on the boundary of $D$, OR
- $P$ is a critical point of $f$.

**Today:** How can we find the maximum value of $f$ along the boundary of $D$, without checking every single point?

# Practice with Lagrange multipliers

We have the following three equations:

$$\begin{aligned} 2x &= \lambda 2x \\ 2y &= -\lambda 2y \\ x^2 + y^2 &= 4. \end{aligned}$$

How many solutions $(x, y, \lambda)$ are there?

(a) No solutions.

(b) 2.

(c) 4.

(d) Infinitely many.

(e) I don't know.

Correct answer: (c)

# Lagrange multipliers (in three variables)

Assume $f, g$ are functions of three variables with continuous first order partial derivatives.

If $f(x_0, y_0, z_0)$ is the maximum value of $f$ over the level surface $g(x, y, z) = k$, then either

- $\nabla g(x_0, y_0, z_0) = \langle 0, 0, 0 \rangle$ OR
- $\nabla f(x_0, y_0, z_0) = \lambda \nabla g(x_0, y_0, z_0)$, for some $\lambda \in \mathbb{R}$.

The same theorems hold for minimum values too.

# Practice with Lagrange multipliers

Suppose $f, g$ are functions on $\mathbb{R}^3$ with continuous first order partial derivatives.

Suppose that $f$ achieves its maximum over the set $\{g(x, y, z) = k\}$ at the point $P$. Which of the following is not possible?

(a) $\nabla f(P) = \langle 0, 0, 0\rangle, \nabla g(P) = \langle 0, 0, 0\rangle$.

(b) $\nabla f(P) = \langle 0, 0, 0\rangle, \nabla g(P) = \langle 1, 3, -2\rangle$.

(c) $\nabla f(P) = \langle 4, 0, 1\rangle, \nabla g(P) = \langle 0, 0, 0\rangle$.

(d) $\nabla f(P) = \langle -2, -6, 4\rangle, \nabla g(P) = \langle 1, 3, -2\rangle$.

(e) $\nabla f(P) = \langle 4, 0, 1\rangle, \nabla g(P) = \langle 1, 3, -2\rangle$.

Correct answer: (e)

# Maximizing the volume of a box

Which of the following is true?

(a) $D$ is closed and bounded, so by the extreme value theorem, $f$ has a maximum and a minimum on $D$.

(b) $D$ is not closed or bounded, so we can't say anything.

(c) $D$ is not closed or bounded, but we can argue for geometric reasons that $f$ has a maximum and a minimum on $D$.

(d) $D$ is not closed or bounded, but we can argue for geometric reasons that $f$ has a maximum but not a minimum.

Correct answer: (d)