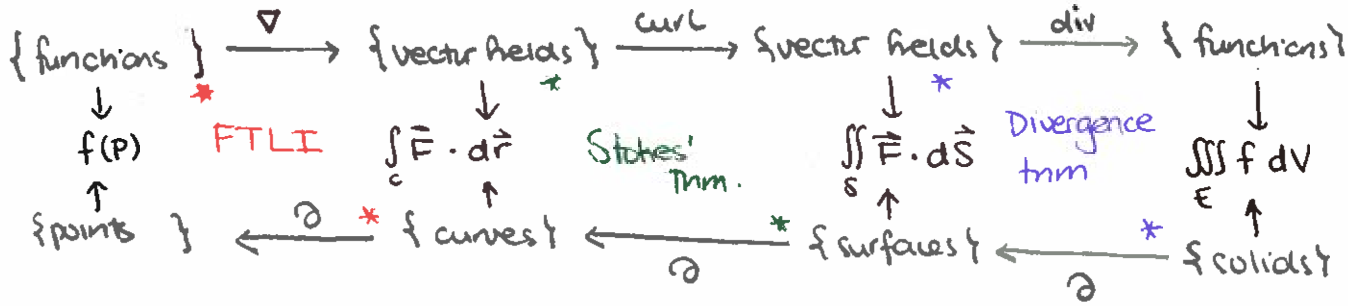
- Last time: topology 101
- Announcements



Fundamental theorem of line integrals.

* Start with a function f and a curve C

1) ∇: $\int_C \nabla f \cdot d\vec{r}$

$\|$

2) ∂: $f(Q) - f(P)$

Stokes' Theorem:

* Start with a vector field $\vec{F}$ and a surface S.

1) curl: $\iint_S \text{curl}\,\vec{F}\cdot d\vec{S}$

$\|$

2) ∂: $\int_{\partial S} \vec{F}\cdot d\vec{r}$

Divergence Theorem:

* Start with a vector field $\vec{F}$ and a solid E.

1) div: $\iiint_E \text{div}\,\vec{F}\ dV$

$\|$

2) ∂: $\iint_{\partial E} \vec{F}\cdot d\vec{S}$

1) Any two consecutive arrows on the top row give 0
2) Any two consecutive arrows on the bottom row give the empty set.
3) Can integrate/evaluate any column to get a number
4) Each square is a theorem

Approaches to solving problems.

$\int_C \vec{F}\cdot d\vec{r}$

(1) if $\vec{F} = \nabla g$, use fundamental theorem of line integrals

(2) solve directly if possible

(3) Are you in $\mathbb{R}^2$ or $\mathbb{R}^3$?

$\mathbb{R}^2$ - use Green's Theorem:

Choose $D \subset \mathbb{R}^2$ so that $\vec{F}$ is nice on D.

(a) ideal: $\partial D = \pm C$

(b) Okay: $\partial D = \pm C \cup C'$, where $\int_{C'} \vec{F}\cdot d\vec{r}$ is "easy"

$\mathbb{R}^3$ - use Stokes' theorem:

Find oriented surface so F is nice on S.

(a) ideal: $\partial S = C$

(b) okay: $\partial S = C \cup C'$ and $\int_{C'} \vec{F}\cdot d\vec{r}$ is "easy".

(4) Approximate: - use $\int_C \vec{F}\cdot d\vec{r} = \int_C \vec{F}\cdot \vec{T}\, ds$

$= (\text{length of } C)(\text{average value of } \vec{F}\cdot\vec{T})$.

- use Riemann sum.

$\iint_S \vec{F}\cdot d\vec{S}$

(1) If $\vec{F} = \text{curl}\,\vec{G}$ is given, use Stokes' theorem.

(2) Solve directly if possible

(3) Use the divergence theorem.

- Find a solid E so that $\vec{F}$ is nice on E

(a) ideal: $\partial E = \pm S$.

(b) okay: $\partial E = \pm S \cup S'$ and $\iint_{S'} \vec{F}\cdot d\vec{S}$ is "easy"

(4) approximate.

What's the best way?

1) Follow instructions

2) Do the easiest problem:

- Squiggles and blobs are hard
- polygons and boxes are hard
- complicated vector fields are hard
- already parametrized surfaces

→ • "approximate" and "positive/negative/zero" are easier

(3) Do steps in order.

* Don't be afraid to waste a little time, but don't waste too much time.