

**Tuesday, April 30** \*\* *Surface integrals of vector fields and related theorems*

1. Consider the region  $D$  in  $\mathbb{R}^3$  bounded by the  $xy$ -plane and the surface  $x^2 + y^2 + z = 1$ .
  - (a) Make a sketch of  $D$ .
  - (b) The boundary of  $D$ , denoted  $\partial D$ , has two parts: the curved top  $S_1$  and the flat bottom  $S_2$ . Parameterize  $S_1$  and calculate the flux of  $\mathbf{F} = (0, 0, z)$  through  $S_1$  with respect to the upward pointing unit normal vector field. Check your answer with the instructor.
  - (c) Without doing the full calculation, determine the flux of  $\mathbf{F}$  through  $S_2$  with the downward pointing normals.
  - (d) Determine the flux of  $\mathbf{F}$  through  $\partial D$  with the outward pointing normals.
  - (e) Apply the Divergence Theorem and your answer in (d) to find the volume of  $D$ . Check your answer with the instructor.
  
2. Consider the vector field  $\mathbf{F} = (-y, x, z)$ .
  - (a) Compute  $\text{curl } \mathbf{F}$ .
  - (b) For the surface  $S_1$  above, evaluate  $\iint_{S_1} (\text{curl } \mathbf{F}) \cdot \mathbf{n} \, dA$ .
  - (c) Check your answer in (b) using Stokes' Theorem.
  
3. If time remains:
  - (a) Check your answer in 1(e) by directly calculating the volume of  $D$ .
  - (b) Repeat 2 (b-c) for the surface  $S_2$  and also for the surface  $\partial D$ . What exactly does 2(c) mean for the surface  $\partial D$ ?
  - (c) For the vector field  $\mathbf{F} = (-y, x, z)$  from the second problem, compute  $\text{div}(\text{curl } \mathbf{F})$ . Now suppose  $\mathbf{F} = (F_1, F_2, F_3)$  is an arbitrary vector field. Can you say anything about the function  $\text{div}(\text{curl } \mathbf{F})$ ?