Mid-Semester Test

Date: 9 September 2011 **Time:** 12:00 **Duration:** 45 minutes

You need to give full justification using results from the lectures, tutes, problem sets and assignments. No calculators or materials are permitted.

The total number of marks on the exam paper is 52.

Q1 (12 marks)

Let $\alpha\colon \mathbb{R} \to \mathbb{R}^3$ be defined by $\alpha(t) = (\cos 2t, 2t, \sin 2t)$.

Find the orthonormal frame (T, N, B), curvature κ and torsion τ as functions of t.

Q2 (8 marks)

Let $\alpha\colon \mathbb{R} \to \mathbb{R}^3$ be given by $\alpha(t) = (2t, t^2, \frac{t^3}{3})$.

Find the arc length function $s = s(t)$ (based at $t = 0$) of α and determine $\text{length}_{[-1,1]}(\alpha)$.

Q3 (10 marks)

Suppose that $\beta = \alpha \circ h\colon J \to \mathbb{R}^n$ is a negative reparametrisation of the curve $\alpha\colon I \to \mathbb{R}^n$ and that α and β are both parametrised by arc length.

Show that $h\colon J \to I$ is defined by $h(u) = -u + u_0$ for some $u_0 \in \mathbb{R}$.

Q4 (7 marks)

State the definition of a regular surface in $\mathbb{R}^3$.

Q5 (15=13+2 marks)

Let $\Phi\colon \mathbb{R}^2 \to \mathbb{R}^3$ be defined by $\Phi(u, v) = (u^3, u + v^3, v)$.

(a) Show that $S = \text{Im}(\Phi)$ is a regular surface.

(b) Is $\Phi\colon \mathbb{R}^2 \to \text{Im}(\Phi)$ a diffeomorphism?

End of exam paper — I hope you enjoy it!