Assignment 2

Due Thursday, 24 March, at 17:00 in the assignment box for MATH3402. The box is located on **Level 3** in the Mathematics (Priestley) building (67). It is in the **white lot of boxes** (there are two lots, to find ours, turn left as you come from the stairs).

Please use a cover sheet!

Clearly state your assumptions and conclusions, and justify all steps in your work.

Marks will be deducted for sloppy working.

Q1 Let $(X, \mathcal{O})$ be a topological space. Let $A, B \subseteq X$.

Give a proof or a counterexample for each of the following statements:

(a) $A \subset B$ implies $A^\circ \subseteq B^\circ$
(b) $A \subset B$ implies $\overline{A} \subseteq \overline{B}$
(c) $A \subset B$ implies $\partial A \subseteq \partial B$
(d) $\overline{A \cup B} = \overline{A} \cup \overline{B}$
(e) $(A \cup B)^\circ = A^\circ \cup B^\circ$
(f) $\overline{A \cap B} = \overline{A} \cap \overline{B}$
(g) $(A \cap B)^\circ = A^\circ \cap B^\circ$

Q2 Let (X, d_X) and (Y, d_Y) be metric spaces. Suppose $f\colon X \to Y$ is a bijection with the property that both f and f^{-1} are uniformly continuous.

Show that (X, d_X) is complete if and only if (Y, d_Y) is complete.

Q3 Let (X, d_X) be a compact metric space, and (Y, d_Y) be an arbitrary metric space.

Show that any continuous function $f\colon X \to Y$ is uniformly continuous.